8/12/2015 الثلاثاء د. عرفة محاضرة [9]

Review: State Space

System

- D.Es (T.F)
- Block diagram or Signal flow graph
- two State eqn
  $$\dot{x} = Ax + Bu$$
  $$y = Cx + Du$$
  time domain
- phisycal Model

For state-space, the system's dynamics in time-domain.

$$\dot{X}(t) = A\,x(t) + B\,\overset{I/P}{u}(t)$$
$$y(t) = C\,x(t) + Du(t)$$

} two state equations.

where: $u(t) \longrightarrow i/p$
$y(t) \longrightarrow o/p$

$\boxed{D = 0} \Leftarrow$ درجة البسط أقل من درجة المقام في ال T.F. phisycal systems

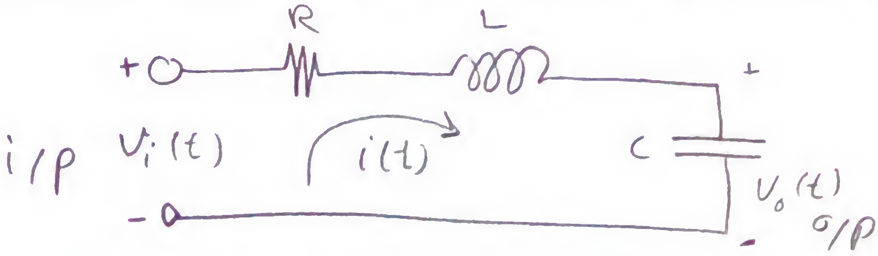
* if D has value, then it would be a scalar (1×1)
* $n \Rightarrow$ system order

- $A_{n\times n} \Longrightarrow$ system matrix
- $B_{n\times 1} \Longrightarrow$ input matrix عمود
- $C_{1\times n} \Longrightarrow$ output matrix صف
- $X(t)_{n\times 1} \Longrightarrow$ state vector $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$

$x_1, x_2, \ldots x_n(t) \Rightarrow$ the states of the system

الـ States تمثل القراءات الفيزيائية المقاسة للنظام

States are the measured values



$i(t) = x_1(t)$ | $\dot{x} = A(t)x + Bu(t)$

$\underbrace{V_o(t)}_{y(t)} = x_2(t)$ | $y(t) = C\,x(t)$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} - & - \\ - & - \end{pmatrix}_{2\times 2} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} - \\ - \end{pmatrix} V_i(t)$$

$$y(t) = \begin{pmatrix} - & - \end{pmatrix}_{1\times 2} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

$* \; V_i(t) = V_R + V_L + V_o$

مو موجود في المعادلة بيضل كما هو

محتاج أجيب التفاضلات $\dot{x}(t)$ من المعادلة

$$= \underbrace{i(t)}_{x_1(t)} R + L \underbrace{\frac{di}{dt}}_{\dot{x}_1(t)} + \underbrace{V_o(t)}_{x_2(t)}$$

$$\Rightarrow \dot{x}_1(t) = -\frac{R}{L} x_1 - \frac{1}{L} x_2 + \frac{1}{L} V_i(t)$$

$$* \; \underbrace{i(t)}_{x_1(t)} = C \underbrace{\frac{dv_o(t)}{dt}}_{\dot{x}_2(t)}$$

$$\dot{x}_2 = \frac{1}{C} x_1$$

$\Longrightarrow$ Turn over

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} U_i(t)$$

$$y(t) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

Canonical Forms for S.S.

$\boxed{1}$ Controllable Form

3rd order system:

$$T.F. = \frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$U(s) \longrightarrow \boxed{\frac{1}{s^3 + a_1 s^2 + a_2 s + a_3}} \xrightarrow{V(s)} \boxed{b_1 s^2 + b_2 s + b_3} \xrightarrow{Y(s)}$$

assume :

$$V = x_1$$

$$\dot{V} = \dot{x}_1 = x_2$$

$$\ddot{V}(t) = \dot{x}_2 = x_3$$

$$\dddot{V}(t) = \dot{x}_3$$

$$\dot{x}_1 = x_2(t)$$

$$\dot{x}_2 = x_3(t)$$

$\dot{x}_3 = - - - -$ ها تهمس ابلول

$$= -a_3 x_1 - a_2 x_2 - a_1 x_3 + u(t)$$

$$\frac{V(s)}{U(s)} = \frac{1}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$(s^3 + a_1 s^2 + a_2 s + a_3) V(s) = U(s) \quad \Downarrow L^{-1}T$$

$$\dddot{V}(t) + a_1 \ddot{V}(t) + a_2 \dot{V}(t) + a_3 V(t) = u(t)$$

$$\frac{Y(s)}{V(s)} = b_1 s^2 + b_2 s + b_3$$

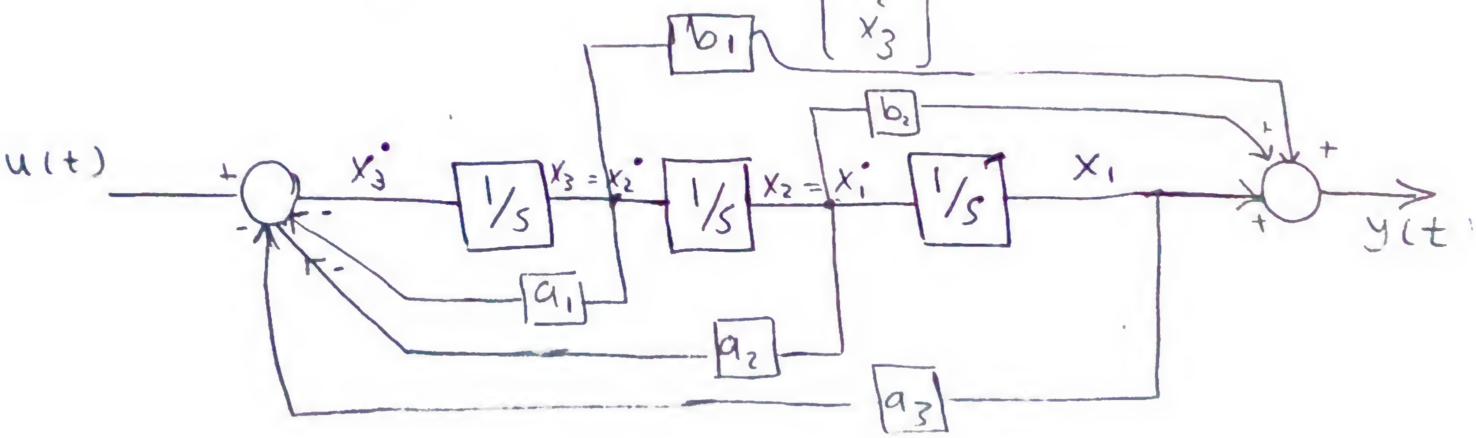
$$Y(s) = (b_1 s^2 + b_2 s + b_3) V(s)$$

$$y(t) = b_1 \underbrace{\ddot{V}(t)}_{x_3} + b_2 \underbrace{\dot{V}(t)}_{x_2} + b_3 \underbrace{V(t)}_{x_1}$$

$$y(t) = b_3 x_1 + b_2 x_2 + b_1 x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} b_3 & b_2 & b_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



to check controllability:

the system is controllable if you can reach any state starting from the input $u(t)$

⟹ Turn Over

معلش ما بتنش كل مرة و أجيب ال Controllable form مباشرة

$$T.F. = \frac{b_1 s^2 + b_2 S + b_3}{(1) S^3 + a_1 S^2 + a_2 S + a_3}$$

معاملها بـ (1)

* عوض مباشرة في الصورة المستنبطة في الصفحة السابقة

* for 4th order System

$$T.F. = \frac{b_1 S^3 + b_2 S^2 + b_3 S + b_4}{S^4 + a_1 S^3 + a_2 S^2 + a_3 S + a_4}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

معاملات المقام معكوسة بإشارة سالبة

$$y(t) = \begin{pmatrix} b_4 & b_3 & b_2 & b_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

معاملات البسط معكوسة

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[2] observable form:

$A_o = A_c^T$ — A_o: observable, A_c: controllable

$B_o = C_c^T$

$C_o = B_c^T$

3rd order System

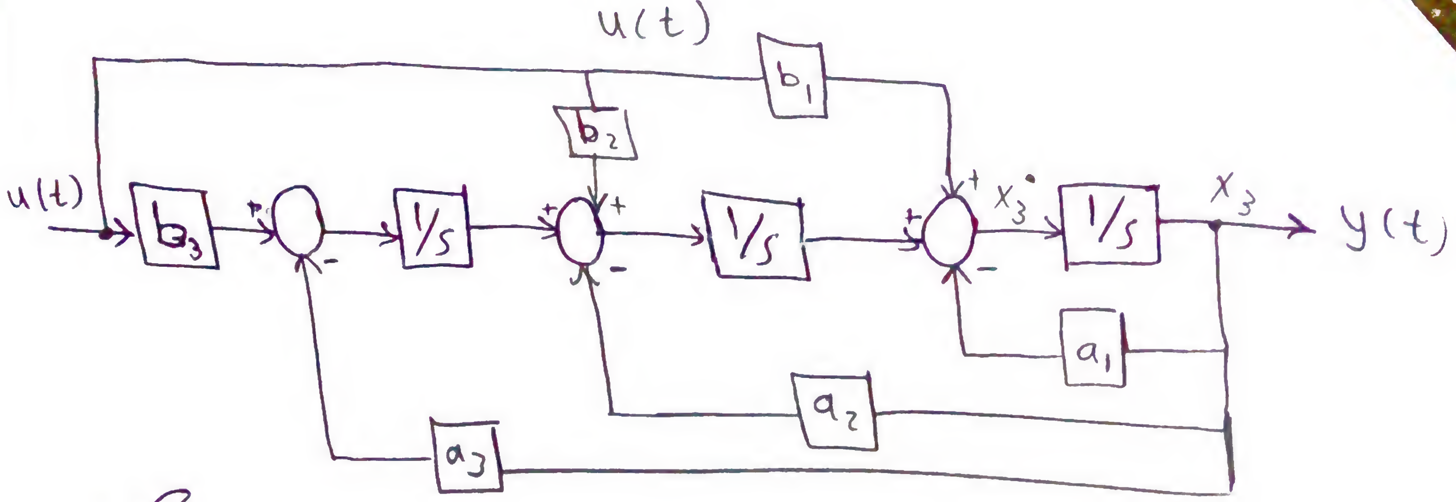
$$T.F. = \frac{b_1 S^2 + b_2 S + b_3}{S^3 + a_1 S^2 + a_2 S + a_3}$$

$$\left[\dot{x}(t)\right] = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} \left[x(t)\right] + \begin{bmatrix} b_3 \\ b_2 \\ b_1 \end{bmatrix} u(t)$$

$\dot{x}_1, \dot{x}_2, \dot{x}_3$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \left[x(t)\right]$$

x_1, x_2, x_3



* for 4th order system

$$T.F. = \frac{b_1 s^3 + b_2 s^2 + b_3 s + b_4}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -a_4 \\ 1 & 0 & 0 & -a_3 \\ 0 & 1 & 0 & -a_2 \\ 0 & 0 & 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Ex: $T.F. = \dfrac{s^2 + 6s + 8}{(s+1)(s+3)(s+5)}$

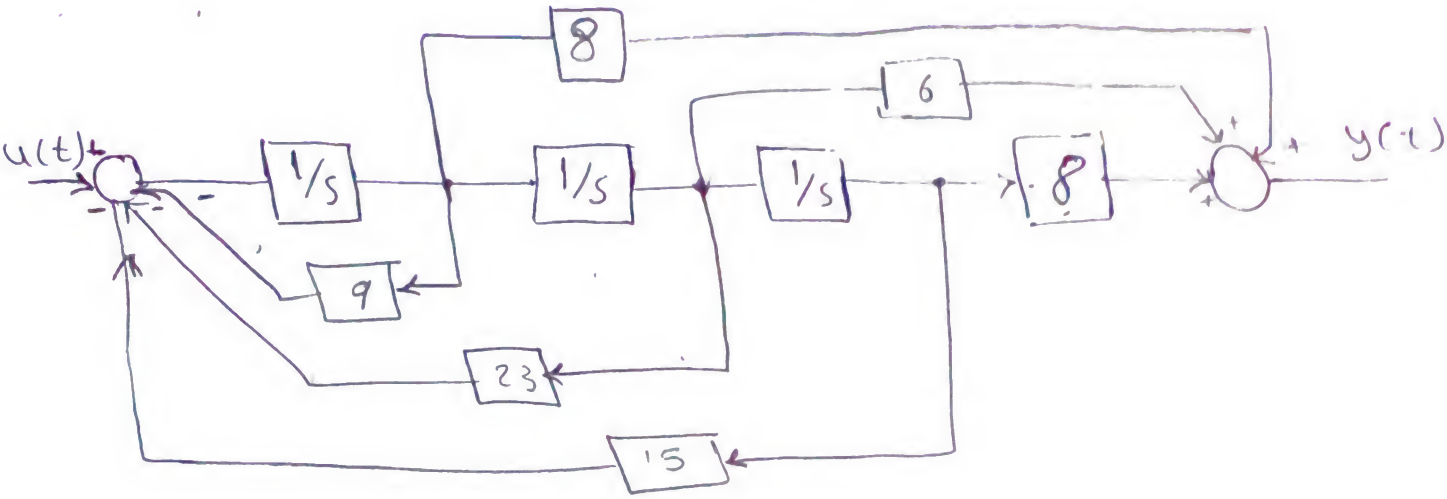
- Find state-space model in controllable and observable forms
- Draw the state diagram for each case.

$$T.F. = \frac{S^2+6S+8}{S^3+9S^2+23S+15}$$

1 Controllable Form:

$$[X^{\cdot}(t)] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -23 & -9 \end{bmatrix} [X(t)] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

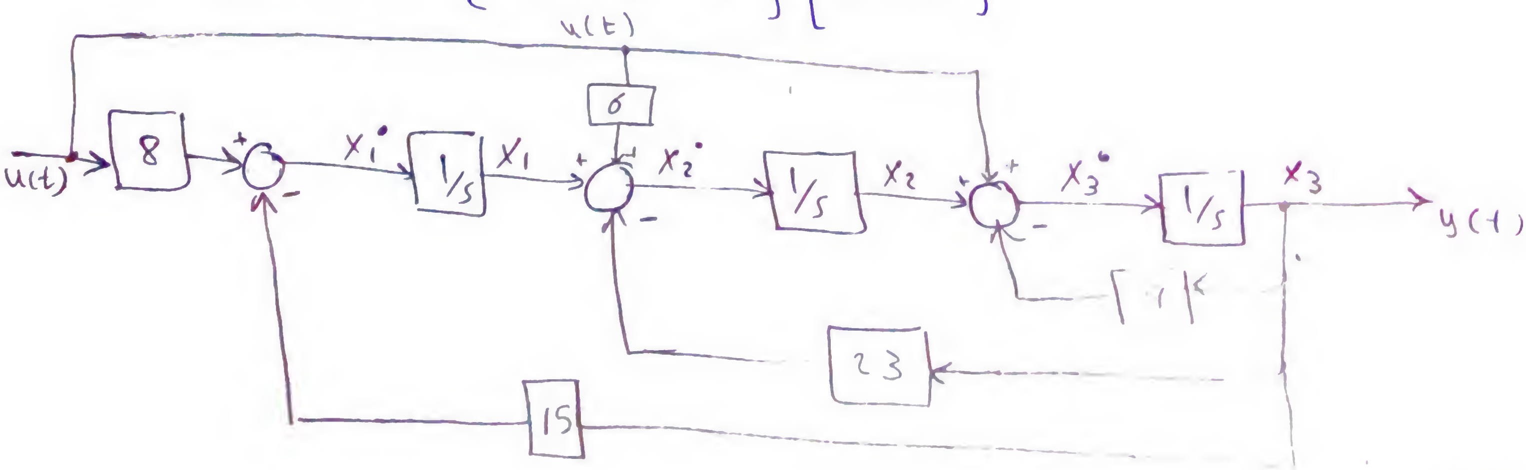
$$y(t) = [\,8 \quad 6 \quad 1\,]\,[X(t)]$$



2 observable form:

$$[X^{\cdot}(t)] = \begin{bmatrix} 0 & 0 & -15 \\ 1 & 0 & -23 \\ 0 & 1 & -9 \end{bmatrix} [X(t)] + \begin{bmatrix} 8 \\ 6 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [\,0 \quad 0 \quad 1\,]\,[X(t)]$$



3 Diagonal Form

for 3rd order system

$$T.F. = \frac{b_1 S^2 + b_2 S + b_3}{S^3 + a_1 S^2 + a_2 S + a_3} \Downarrow$$

$$= \frac{b_1 S^2 + b_2 s + b_3}{(S+P_1)(S+P_2)(S+P_3)} \Downarrow P.F.$$

$$T.F. = \frac{Y(s)}{U(s)} = \frac{A_1}{S+P_1} + \frac{A_2}{S+P_2} + \frac{A_3}{s+P_3}$$

$$Y(s) = \underbrace{\frac{U(s)}{(S+P_1)}}_{X_1(S)} A_1 + \underbrace{\frac{U(s)}{(S+P_2)}}_{X_2(s)} A_2 + \underbrace{\frac{U(s)}{(S+P_3)}}_{X_3(S)} A_3$$

$$X_1(S) = \frac{U(S)}{S+P_1}$$

$$U(s) = (S+P) X_1(S) \downarrow L^{-1}T$$

$$u(t) = \dot{X}_1(t) + P_1 X_1(t) \Rightarrow$$

$$\dot{X}_1 = -P_1 X_1 + u(t)$$

$$\dot{X}_2 = -P_2 X_2 + u(t)$$

$$\dot{X}_3 = -P_3 X_3 + u(t)$$

$$[\dot{X}(t)] = \begin{bmatrix} -P_1 & 0 & 0 \\ 0 & -P_2 & 0 \\ 0 & 0 & -P_3 \end{bmatrix} [X(t)] + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$Y(s) = A_1 X_1(S) + A_2 X_2(S) + A_3 X_3(S)$$

$$y(t) = A_1 X_1(t) + A_2 X_2(t) + A_3 X_3(t)$$

$$y(t) = [A_1 \quad A_2 \quad A_3][X(t)]$$

assuming you got repeated poles

Poles $\longrightarrow -P_1, -P_1, -P_2$ $\qquad T.F. = \dfrac{b_1 s^2 + b_2 s + b_3}{(s+P_1)^2 (s+P_2)}$

$$Y(s) = \frac{U(s) A_1}{(s+P_1)^2} + \frac{U(s) A_2}{(s+P_1)} + \frac{U(s) A_3}{(s+P_2)}$$

$$X_1(s) = \frac{U(s)}{(s+P_1)^2} = \underbrace{\frac{U(s)}{(s+P_1)}}_{X_2} \frac{1}{(s+P_1)}$$

$$X_1(s) = \frac{X_2(s)}{(s+P_1)} \Longrightarrow \dot{x}_1 = -P_1 x_1 + x_2$$

$$X_2(s) = \frac{U(s)}{(s+P_1)} \Rightarrow \dot{x}_2 = -P_1 x_2 + u(t)$$

$$\dot{x}_3 = -P_2 x_3 + u(t)$$

$$[\dot{x}(t)] = \begin{bmatrix} -P_1 & 1 & 0 \\ 0 & -P_1 & 0 \\ 0 & 0 & -P_2 \end{bmatrix} [x(t)] + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix} [x(t)]$$

For $T.F. = \dfrac{b_1 s^2 + b_2 s + b_3 s}{(s+P_1)^3}$

$$[\dot{x}(t)] = \begin{bmatrix} -P_1 & 1 & 0 \\ 0 & -P_1 & 1 \\ 0 & 0 & -P_1 \end{bmatrix} [x(t)] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix} [x(t)]$$

Ex: $TF = \dfrac{s^2+6s+8}{(s+1)(s+3)(s+5)}$

- Find the state space in diagonal form
- Draw the state diagram

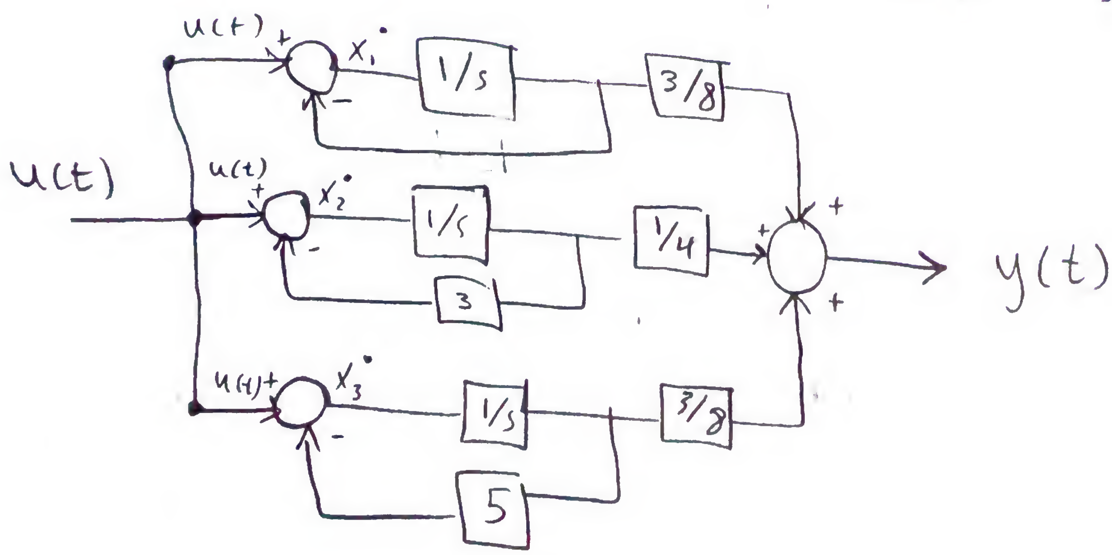
$$T.F. = \frac{s^2+6s+8}{(s+1)(s+3)(s+5)}$$

$$= \frac{A_1}{(s+1)} + \frac{A_2}{(s+3)} + \frac{A_3}{(s+5)}$$

$A_1 = \dfrac{1-6+8}{(2)(4)} = \dfrac{3}{8}$; $A_2 = \dfrac{9-18+8}{(-2)(2)} = \dfrac{-1}{-4} = \dfrac{1}{4}$; $A_3 = \dfrac{25-30+8}{(-4)(-2)} = \dfrac{3}{8}$

$$[\dot{x}(t)] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix} [x(t)] + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 3/8 & 1/4 & 3/8 \end{bmatrix} [x(t)]$$



Ex: $T.F. = \dfrac{2s^2+6s+5}{(s+1)^2(s+2)}$ $\Downarrow$ P.F.

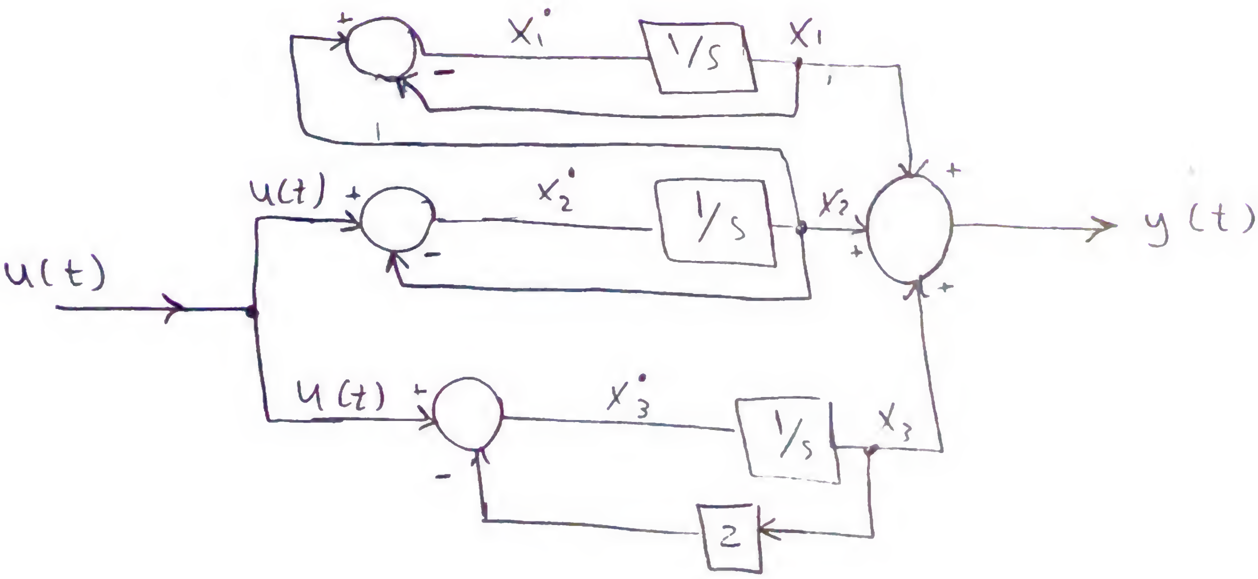
$$T.F. = \frac{A_1}{(s+1)^2} + \frac{A_2}{(s+1)} + \frac{A_3}{(s+2)}$$

$A_1 = \dfrac{2-6+5}{(-1+2)} = 1$; $A_3 = \dfrac{8-12+5}{1} = 1$;

for A_2
put $s=0$
$\frac{5}{2} = A_1 + A_2 + A_3/2$
$= 1 + A_2 + 1/2$
$A_2 = 1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



$\Longrightarrow$ Turn Over

* State Space Analysis

given the two state equations

$$\dot{x} = A x + B u$$
$$y = C x$$

1. T.F.

$$S\,X(S) = A\,X(S) + B\,U(S) \longrightarrow (1)$$
$$Y(S) = C\,X(S) \longrightarrow (2)$$

two state eqns in S-domain

$$S\,X(S) - A\,X(S) = B\,U(S)$$
$$(SI - A)\,X(S) = B\,U(S)$$

$$\Longrightarrow X(S) = (SI - A)^{-1} B\,U(S) \quad (3)$$

where $I_{n \times n}$ is identity matrix

$$Y(S) = C(SI - A)^{-1} B\,U(S) \quad (4)$$

$$T.F. = \frac{Y(S)}{U(S)} = C\,(SI - A)^{-1} B$$

2. Ch. eq.:

$$|SI - A| = 0$$

The roots of ch. eqn = Poles = eigen values

3. The system response to i/p $u(t)$

o/p in time domain

if $X(0) \neq 0$

⟹ Turn over

if $x(0) \neq 0$

$$\dot{x}(t) = A x(t) + B u(t) \xrightarrow{LT} S X(s) - X(0) = A X(s) + B u(s)$$

$$S X(S) - A X(S) = X(0) + B u(S)$$

$$(S I - A) X(S) = X(0) + B u(S)$$

$$X(S) = (SI - A)^{-1} X(0) + (SI - A)^{-1} B u(S)$$

Let $\phi(S) = (SI - A)^{-1} \Rightarrow$ Transition Matrix

$$X(S) = \phi(S) X(0) + \phi(S) B u(S)$$

$$y(t) = C x(t)$$

$$Y(S) = C X(S)$$

$$= C [\phi(S) X(0) + \phi(S) B u(S)]$$

$$Y(s) \xrightarrow{L^{-1}T} y(t)$$

EX: $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$

$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$ & $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Find T.F. & $y(t)$ for unit-Step response

Solution:-

$$T.F. = C (SI - A)^{-1} B$$

$$*(SI - A) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} s & -1 \\ 2 & s+3 \end{pmatrix}$$

$$*(SI - A)^{-1} = \frac{1}{s(s+3)} \begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}$$

$$*\ (SI-A)^{-1}=\frac{1}{S^2+3S+2}\begin{pmatrix} S+3 & 1 \\ -2 & S\end{pmatrix}$$

$$T.F.=C\,(SI-A)^{-1}\,B$$

$$=\begin{bmatrix}0 & 1\end{bmatrix} * \frac{1}{S^2+3S+2}\begin{pmatrix} S+3 & 1 \\ -2 & S\end{pmatrix}\begin{pmatrix}0 \\ 2\end{pmatrix}$$

$$=\frac{1}{S^2+3S+2}\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix} S+3 & 1 \\ -2 & S\end{pmatrix}\begin{pmatrix}0 \\ 2\end{pmatrix}$$

$$=\frac{1}{S^2+3S+2}\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}2 \\ 2S\end{pmatrix}$$

$$=\frac{2S}{S^2+3S+2}$$

Ch. eqn : $|SI-A|=0$ محسوب سابقا

$$S^2+3S+2=0$$

$$X(s)=\phi(s)\,X(0)+\phi(s)\,B\,u(s)$$

$$\phi(s)=(SI-A)^{-1}=\frac{1}{S^2+3S+2}\begin{pmatrix} S+3 & 1 \\ -2 & S\end{pmatrix}$$

$u(t)=1$

$U(s)=1/s$

$$X(s)=\frac{1}{(S^2+3S+2)}\begin{pmatrix} S+3 & 1 \\ -2 & S\end{pmatrix}\begin{pmatrix}0 \\ 1\end{pmatrix}$$

$$+\ \frac{1}{(S^2+3S+2)}\begin{pmatrix} S+3 & 1 \\ -2 & S\end{pmatrix}\begin{pmatrix}0 \\ \cancel{2}\end{pmatrix}\left(\frac{1}{S}\right)$$

$2/S$

$$= \frac{1}{s^2+3s+2}\left[\begin{pmatrix}1\\ s\end{pmatrix}+\begin{pmatrix}2/s\\ 2\end{pmatrix}\right]$$

$$= \frac{1}{s^2+3s+2}\begin{pmatrix}1+\frac{2}{s}\\ s+2\end{pmatrix}$$

$$X(s) = \frac{1}{(s+1)(s+2)}\begin{pmatrix}\frac{s+2}{s}\\ s+2\end{pmatrix} = \begin{pmatrix}\frac{1}{s(s+1)}\\ \frac{1}{s+1}\end{pmatrix}$$

$$Y(s) = C\,X(s)$$

$$= \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{s(s+1)}\\ \frac{1}{s+1}\end{pmatrix}$$

$$Y(s) = \frac{1}{s+1} \Rightarrow L^{-1}T \quad y(t) = e^{-t}$$

unit-Step Response

4 Controllability

* The system is completely controllable if the system states can be changed by changing the system i/p

* Another definition :-

The ability of control i/P signal of a system to move any initial state to another final states during finite intervals of time

$$x(t_0) \longrightarrow x(t_1)$$

Controllability Matrix (M_c)

$$M_c = (B \quad AB \quad A^2B \cdots\cdots A^{n-1}B)$$

if $|M_c| \neq 0$, the system is controllable.

* 2nd order $\Rightarrow M_c = (B \quad AB)$

* 3rd order $\Rightarrow M_c = (B \quad AB \quad A^2B)$ — $n-1$

[5] observability

* In some cases the states connot be measured for the following reasons
  1- The location for phisycal states :-
  2- The measuring instruments are not valid.
  in this case, an estimation for these states is required
* If the internal states of a system could be estimated (calculated) from the observation of o/p response, then the system is called observable.

observability matrix (M_o)

$$M_o = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{pmatrix}$$

$$|M_o| \neq 0 \Rightarrow \text{observable}$$

$M_o = \begin{pmatrix} C \\ CA \end{pmatrix} \rightarrow$ 2nd order

$M_o = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} \rightarrow$ 3rd order

Ex: $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$

$M_c = \begin{pmatrix} B & AB & A^2B \end{pmatrix}$

$AB = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $A^2B = A.AB = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

$M_c = \begin{pmatrix} +0 & 1 & 2 \\ -1 & 1 & 0 \\ +0 & 1 & 2 \end{pmatrix} \Rightarrow |M_c| = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$

the system is not controllable

$M_o = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix}$; $CA = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$

$CA^2 = CA.A = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 4 & -1 \end{pmatrix}$

$M_o = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 4 & -1 \end{pmatrix} \Rightarrow |M_o| = \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}$

$= -6 + 2 = -4 \neq 0$

the system is observable